Name: ____________________

Teacher: ____________________

Class: ____________________

FORT STREET HIGH SCHOOL

2015

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics Extension 1

Time allowed: 2 hours
(plus 5 minutes reading time)

Syllabus Outcomes	Assessment Area Description and Marking Guidelines	Questions
	Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
HE2, HE4	Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry, polynomials and circle geometry.	11, 12
HE3, HE5 HE6	Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	13
HE7	Synthesises mathematical solutions to harder problems and communicates them in appropriate form	14

Total Marks 70

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 60 Marks
Attempt Questions 11-14,
Allow about 1 hour 45 minutes for this section

General Instructions:

- Questions 11-14 are to be started in a new booklet.
- The marks allocated for each question are indicated.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations.
- Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used.

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 60	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
	Percent	

SECTION I (One mark each)

Answer each question by circling the letter for the correct alternative on this sheet.

Allow about 15 minutes for this section.

1 Which expression is a correct factorisation of $x^3 + 64$

(A) $(x-4)\,(x^2-4x+16)$

(B) $(x-4)\,(x^2-8x-16)$

(C) $(x+4)\,(x^2+4x-16)$

(D) $(x+4)\,(x^2-4x+16)$

2 Which expression is equal to $\int sin^2\, 3x\, dx$?

(A) $\frac{1}{2}\,(\,x - \frac{1}{3}\sin 3x\,) + C$

(B) $\frac{1}{2}\,(\,x + \frac{1}{3}\sin 3x\,) + C$

(C) $\frac{1}{2}\,(\,x - \frac{1}{6}\sin 6x\,) + C$

(D) $\frac{1}{2}\,(\,x + \frac{1}{6}\sin 6x\,) + C$

3 Which inequality has the same solutions as

$|\,x+2\,| + |\,x-3\,| = 5$?

(A) $x^2 - x - 6 \le 0$

(B) $\frac{1}{x-3} - \frac{1}{x+2} \le 0$

(C) $|\,2x-1| \ge 5$

(D) $\frac{5}{3-x} \ge 1$

4 A Mathematics department consists of 5 female and 5 male teachers. How many committees of 3 teachers can be chosen which contain at least one female and one male?

(A) 100

(B) 120

(C) 200

(D) 2500

5 Consider the function $f(x) = \frac{2x}{x+1}$ and its inverse function $f^{-1}(x)$. Evaluate $f^{-1}\,(3)$.

(A) -3

(B) $\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 3

6 Which group of three numbers could be the roots of the polynomial equation

$$x^3 + a\,x^2 - 41\,x + 42 = 0 \text{ ?}$$

(A) 2, 3 , 7

(B) 1, -6, 7

(C) -1, -2, 21

(D) -1, -3, -14

7 A family of ten people is seated randomly around a circular table. What is the probability that the two oldest members of the family sit together?

(A) $\frac{2!6!}{10!}$

(B) $\frac{8!2!}{10!}$

(C) $\frac{8!2!}{9!}$

(D) $\frac{9!2!}{9!}$

8) Let $x = 1$ be a first approximation to the root of the equation $\cos x = \log_e x$.
What is a better approximation to the root using Newton's method?

(A) 1.28

(B) 1.29

(C) 130

(D) 1.31

9 What is the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sec^2 x}{\tan x}\,dx$? Use the substitution $u = \tan x$.

(A) -0.6009

(B) 0.6913

(C) $\log_e \sqrt{3}$

(D) $\log_e 3$

10 Let $|a| \leq 1$. What is the general solution of $Sin\ 2x = a$?

(A) $x = n\pi + (-1)^n \frac{sin^{-1} a}{2}$, n is an integer

(B) $x = \frac{n\pi + (-1)^n sin^{-1} a}{2}$, n is an integer

(C) $x = \frac{2n\pi \pm sin^{-1} a}{2}$, n is an integer

(D) $x = 2n\pi \pm \frac{sin^{-1} a}{2}$, n is an integer

Question 11 (15 marks) Use a NEW writing booklet.

a) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{5x}$ 1

b) Find $\int \frac{x+1}{x^2+4}\, dx$ 2

c) Find $\frac{d}{dx}\left[cos^{-1}(3x^2)\right]$ 2

d) Find the acute angle between the lines $3y = 2x + 8$, and $y = 5x - 9$. 2

e) The points $P\ (2ap, ap^2)$ and $Q\ (2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) The equation of the chord PQ is $y = \frac{1}{2}(p+q)x - apq$. (Do NOT prove this).

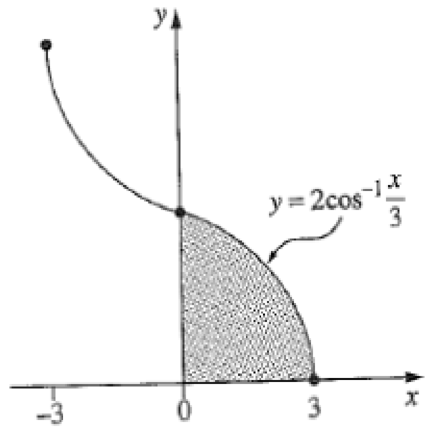
If the chord PQ passes through$(0, a)$, show that $pq = -1$. 1

(ii) Given the chord PQ passes through $(0, a)$ and the normals at P and Q intersect at the point R whose coordinates are

$$(-apq\,[p+q],\, a\,[p^2 + pq + q^2 + 2]) .$$

Find the equation of the locus of R. 2

f)



The sketch shows the graph of the curve $y = f(x)$ where $f(x) = 2\cos^{-1}\frac{x}{3}$.

The area under the curve for $0 \leq x \leq 3$ is shaded.

(i) Find the y intercept. 1

(ii) Find the domain and range of $f(x) = 2\cos^{-1}\frac{x}{3}$. 2

(iii) Calculate the area of the shaded region. 2

Question 12 (15 marks) Use a NEW writing booklet

a) Let α, β, γ be the roots of the equation $x^3 - 3x^2 - 6x - 1 = 0$.

(i) Find $2\alpha + 2\beta + 2\gamma$. 1

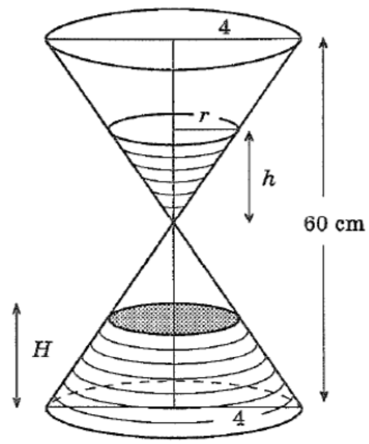
(ii) Find $\alpha^2 + \beta^2 + \gamma^2$ 2

b) A particle moves in a straight line and its position in metres at anytime t seconds is given by $x = 3\cos 2t - 4\sin 2t$

(i) Express the motion in terms of $A\cos(nt + \alpha)$. 2

(ii) Find the particle's greatest speed. (Answer to the nearest whole number). 2

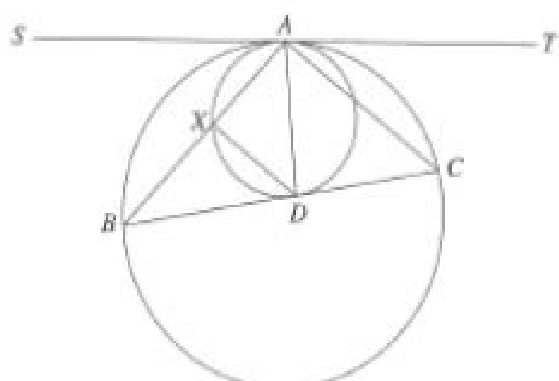
c) A coffee maker has the shape of a double cone 60cm high. The radii at both ends are 4cm. Coffee is flowing from the top cone at the rate of $5cm^3/s$.



(i) Show that radius (R) in the bottom cone is $\frac{2\ (30-H)}{15}$ 1

(ii) How fast is the level of coffee in the bottom cone rising at the instant when the coffee in this cone is 6 cm deep? 3

d) In the diagram, ST is tangent to both the circles at A.
The points B and C are on the larger cicles, and the line BC is a tangent to the smaller circle at D. The line AB intersects the smaller circle at X.



Copy or trace the diagram into your answer booklet.

i) Explain why $\angle AXD = \angle ABD + \angle XDB$ 1
ii) Explain why $\angle AXD = \angle TAC + \angle CAD$ 1
iii) Hence show that AD bisects $\angle BAC$ 2

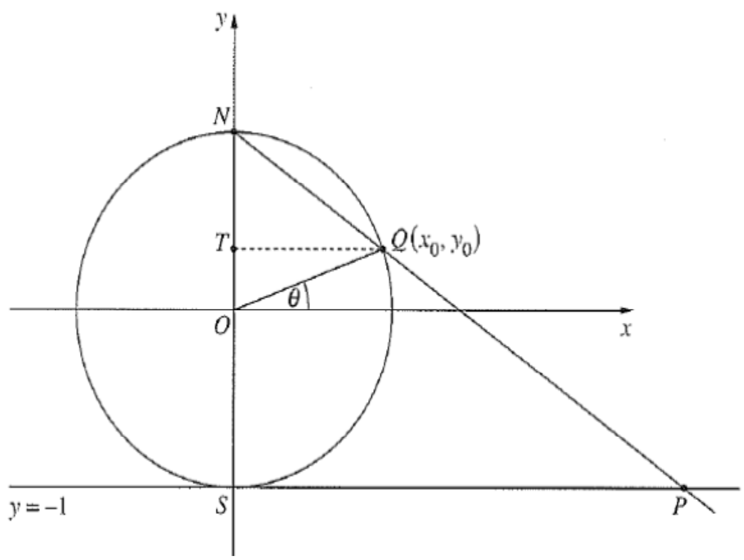
Question 13 (15 marks) Use a NEW writing booklet

a) In a bag there are 6 red, 4 white and 3 black balls. Three balls are drawn simultaneously. What is the probability that these are:

(i) all red. **1**

(ii) exactly 2 white balls. **1**

b) In the diagram, $Q\ (\ x_0\ , y_0\ )$ is a point on the unit circle $x^2 +\ y^2 = 1$ at an angle θ from the positive x –axis, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The line through $N\ (0,1)$ and Q intersects the line $y = -1$ at P. The points $T\ (\ 0\ , y_0\ )$ and $S\ (\ 0\ , -1)$ are on the y –axis.



(i) Using the fact that $\Delta\, TQN$ and $\Delta\, SPN$ are similar,

show that $SP = \frac{2\ \cos\theta}{1-\sin\theta}$ **2**

(ii) Show that $\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$. **1**

(iii) Show that $\angle SNP = \frac{\theta}{2} + \frac{\pi}{4}$ **1**

(iv) Hence, show that $\tan(\frac{\theta}{2} + \frac{\pi}{4}) = \sec\theta + \tan\theta$. **1**

c) A freshly caught fish, initially at 18°C, is placed in a freezer that has a constant

unknown temperature of $x°C$. The cooling rate of the fish is proportional to the difference between the temperature of the freezer and the temperature $T°\ C$, of the fish.

It is known that T satisfies the equation $\frac{dT}{dt} = -k\,(T - x)$,

where t is the number of minutes after the fish is placed in the freezer.

(i) Show that $T = x + Ae^{-kt}$ satisfies this equation. 1

(ii) If the temperature of the fish is $10°\ C$ after $7\frac{1}{2}$ minutes,

Show that the fish's temperature after t minutes is given by 3

$$T = x + (18 - x)e^{\frac{2}{15}\log_e\left[\frac{10-x}{18-x}\right]t}$$

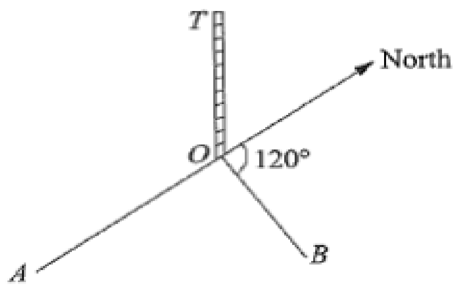
(iii) Find the temperature of the fish after 15 minutes when the initial freezer temperature is $5°C$. Answer to the nearest degree. 1

d) Use the principle of mathematical induction to show that 3

$4^n - 1 - 7n > 0$ for all integers $n \geq 2$.

Question 14 (15 marks) Use a NEW writing booklet

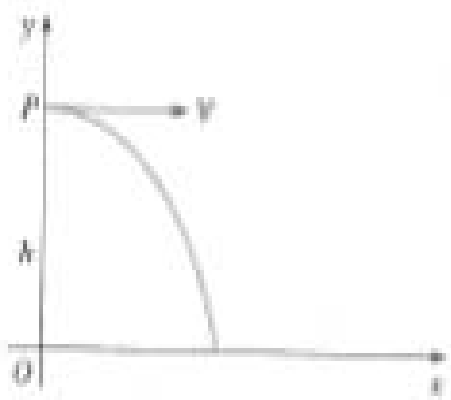
a) From a point A is due south of a tower, the angle of elevation of the top of the tower T, is $23°$. From another point B, on a bearing of $120°$, from the tower, the angle of elevation of T is $32°$. The distance AB is 200 metres.



i) Copy or trace the diagram into your writing booklet, adding the given information to your diagram. 1

ii) Hence find the height of the tower to the nearest metre. 3

b) A particle is projected horizontally from a point P, h metres above O, with a velocity of V metres per second. The equation of the motion of the particle are
$\ddot{x} = 0$ and $\ddot{y} = -g$.



(i) If the horizontal position of the particle at time t is $x = Vt$, show that the vertical position is given by 1
$y = h - \frac{1}{2} g t^2$.

A canister containing a life raft is dropped from a helicopter to a stranded sailor. The helicopter is travelling at a constant velocity of 216 km/h, at a height of 120 metres above sea level, along a path that passes above the sailor.

(ii) How long will the canister take to hit the water? (Answer to one decimal place). (Take $g = 10\ m/s^2$). 2

(iii) A current is causing the sailor to drift at a speed of 3.6 km/h in the same direction as the plane is travelling. The canister is dropped from the plane when the horizontal distance from the plane to the sailor is D metres. What values can D take if the canister lands at most 50 metres from the stranded sailor? 4

c) The depth of water y metres on a tidal creek is given $y = 5 - 4\cos\frac{t}{2}$, for $0 \leq t \leq 4\pi$. the time being measured in hours.

(i) Draw a neat sketch of 2
$y = 5 - 4\cos\frac{t}{2}$, showing all important features.

(ii) If the low tide one day is at 1.00 p.m., when is the earliest time that a ship requiring 3 m of water can enter the creek? Give your answer in hours and minutes.
2

END

Teacher's Solution.

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3 Which inequality has the same solutions as
$|x+2|+|x-3|=5$?

(A) $x^2-x-6\le 0$
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(A) -3
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) 3

6 Which group of three numbers could be the roots of the polynomial equation
$x^3+ax^2-41x+42=0$?

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(D) $x=2n\pi\pm\frac{\sin^{-1}a}{2}$, n is an integer

Solutions | comments

Question 11

a) $\lim_{x \to 0} \frac{\sin 3x}{5x} = \lim_{x \to 0} \frac{3}{5} \frac{\sin 3x}{3x}$

$= \frac{3}{5} \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= \frac{3}{5}$ ✓

well done

b) $\int \frac{x+1}{x^2+4}\,dx = \int \frac{x}{x^2+4} + \frac{1}{x^2+4}\,dx$

$= \frac{1}{2} \int \frac{2x}{x^2+4}\,dx + \frac{1}{2} \int \frac{2}{x^2+4}\,dx$

$= \frac{1}{2} \ln(x^2+4) + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$ ✓ ✓

well done.

c) $\frac{d}{dx}\left(\cos^{-1}(3x^2)\right) = -\frac{1}{\sqrt{1-(3x^2)^2}} \cdot 6x$ ✓

$= \frac{-6x}{\sqrt{1-9x^4}}$ ✓

d) $y_1 = \frac{2x}{3} + \frac{8}{3} \quad \therefore m_1 = \frac{2}{3}$

$y_2 = 5x - 9 \quad \therefore m_2 = 5$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\frac{5 - \frac{2}{3}}{1 + \frac{10}{3}}\right|$ ✓ $\quad \therefore \theta = 45^\circ$ ✓

$= 1$

good.

e)

i) $y = \frac{1}{2}(p+q)x - apq$ passes through $(0, a)$

$a = 0 - apq$ ✓

good.

$pq = -\frac{a}{a}$

$\therefore pq = -1$

ii The equation of the locus:

$x = -apq(p+q)$ — ①

$y = a(p^2+pq+q^2+2)$ — ②

from ① $pq = -1$

$x = a(p+q)$

$\therefore (p+q) = \frac{x}{a}$ — ③

and $(p+q)^2 = p^2 + 2pq + q^2$
$= p^2 + q^2 - 2$

$\therefore (p+q)^2 + 3 = p^2 + q^2 + 1$ ✓

many students did not get to this stage of the solutions

from ② $y = a(p^2+pq+q^2+2)$

$= a(p^2+q^2+1)$

$= a(p+q)^2 + 3$

$= a\left[\left(\frac{x}{a}\right)^2 + 3\right]$

$= a\left(\frac{x^2+3a^2}{a^2}\right)$

$ay = x^2 + 3a^2$

$\therefore x^2 = a(y-3a)$ ✓

Many students lost a mark for not putting the final answer back to ~~this~~ this form.

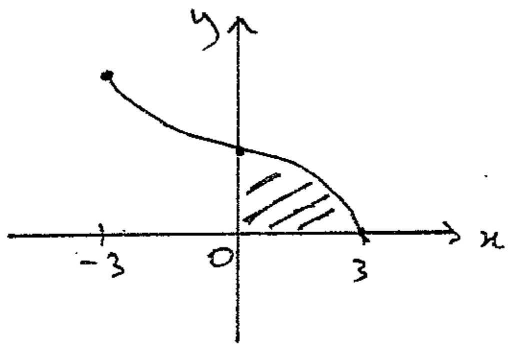
2

f) i) y-intercept : $x = 0$

$y = 2\cos^{-1} 0$

$= 2 \times \frac{\pi}{2}$

$= \pi$ ✓



well done.

ii Domain : $-1 \le \frac{x}{3} \le 1$

$-3 \le x \le 3$ ✓

Range : $0 \le \frac{y}{2} \le \pi$

$0 \le y \le 2\pi$ ✓

Most got this right.

iii Shaded Area $= \int_0^{\pi} 3\cos\frac{y}{2}\,dy$

$y = 2\cos^{-1}\frac{x}{3}$

$\cos^{-1}\frac{x}{3} = \frac{y}{2}$

$x = 3\cos\frac{y}{2}$

$= 3\int_0^{\pi} \cos\frac{y}{2}\,dy$

$= 6\left[\sin\frac{y}{2}\right]_0^{\pi}$ ✓

$= 6\sin\frac{\pi}{2}$

$= 6$ units^2 ✓

Many students lot a mark for not being able to do the integration correctly.

Question 12

a) $x^3 - 3x^2 - 6x - 1 = 0$

i) $2\alpha + 2\beta + 2\gamma = 2(\alpha+\beta+\gamma)$

$= 2\left(-\frac{b}{a}\right)$

$= 6$ ✓

well done

ii $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ ✓ well done

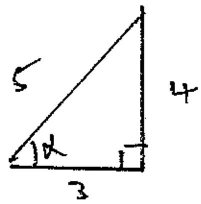
$= (3)^2 - 2\left(\frac{c}{a}\right)$

$= 9 - 2(-6)$

$= 21$ ✓

b) i $3\cos 2t - 4\sin 2t = A\cos(nt+\alpha)$

$= 5\cos\left[2t + \tan^{-1}\left(\frac{4}{3}\right)\right]$ ✓



OR $x = 5\cos(2t + 52^\circ)$ well done.

or $x = 5\cos(2t + 0.927)$

$\tan\alpha = \frac{4}{3}$ ✓

ii The greatest speed when $x = 0$, $\ddot{x} = 0$

$5\cos(2t + 0.927) = 0$

$2t + 0.927 = \frac{\pi}{2}$

$t = 0.32$ sec (2 d.p) ✓ well done

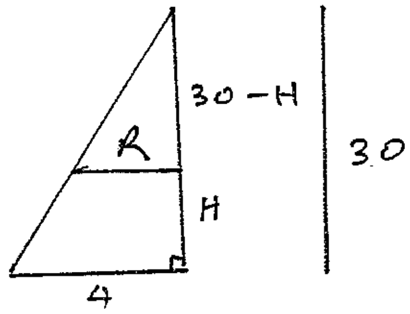
$\therefore \quad \dot{x} = -10\sin(2t + 0.927)$

$= -10\sin(2\times 0.32 + 0.927)$

$\simeq -10$ m/s ✓

$\dot{x} \simeq 10$ m/s moving to the left.

i



$$\frac{R}{4} = \frac{30-H}{30} \checkmark$$

$$R = \frac{4(30-H)}{30}$$

$$R = \frac{2}{15}(30-H)$$

although a simple exercise, many students had a great deal of difficulty.

i

$\frac{dV}{dt} = 5 \text{ cm}^3/\text{s}$

$H = 6$

$$V = \pi R^2 H$$

$$= \frac{\pi}{3} \cdot 4^2 \cdot 30 - \frac{\pi}{3}\left[\frac{2}{15}(30-H)\right]^2(30-H)$$

$$= 160\pi - \frac{4\pi}{675}(30-H)^3 \checkmark$$

$$\frac{dV}{dH} = \frac{3 \times 4\pi}{675}(30-H)^2 \qquad (H=6)$$

$$= \frac{6912\pi}{675} \checkmark$$

$$\therefore \frac{dH}{dt} = \frac{dV}{dt} \times \frac{dH}{dV}$$

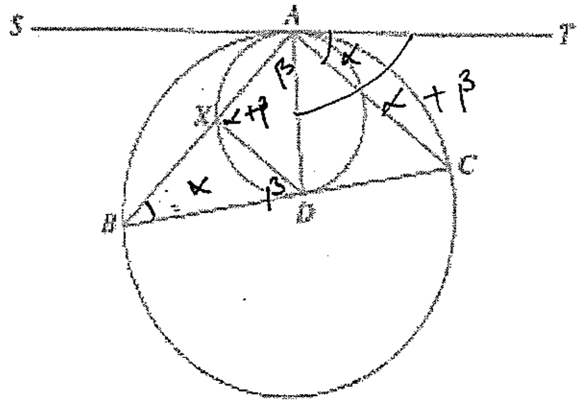
$$= 5 \times \frac{675}{6912\pi}$$

$$= \frac{125}{256\pi} \text{ cm/s} \checkmark$$

$$= 0.1554 \text{ cm/s } (4 \text{ dp}).$$

Few students were able to fully succeed with this question. Many failed to realise that the required volume was the difference between two cones. Confusion in labelling variable lengths also added to their difficulty.

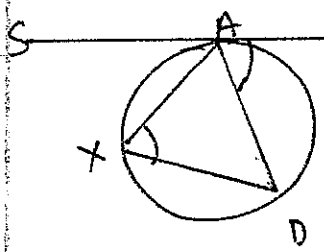
d



i $\angle AXD = \angle ABD + \angle XDB$

$\angle AXD$ is an exterior angle of ΔBDX, the exterior angle is equal to the sum of the opposite interior angles. — well done

ii $\angle AXD = \angle TAD$



An angle between a tangent and a chord is equal to the angle in the alternate segment. — well done.

iii let $\angle XBD = \alpha$ and $\angle XDB = \beta$

$\therefore \angle AXD = \alpha + \beta$ part (i)

Hence $\angle TAC = \angle ABC = \alpha$ (alternate ✓ Segment theorem)

Also $\angle AXD = \angle TAD = \alpha + \beta$ from ii

$\therefore \angle CAD = \alpha + \beta - \alpha$
$= \beta$

and $\angle XAD = \angle XDB = \beta$ ✓ (alternate segment theorem)

$\therefore \angle CAD = \angle XAD = \beta$

Hence, AD bisects $\angle BAC$

often not attempted or students unable to complete this part.

6

Question 13

	Marking	Comments
a) $6R,\ 4W,\ 3B = 13$ total		
(i) $P(\text{all } R) = \dfrac{{}^6C_3}{{}^{13}C_3} = \dfrac{10}{143}$	① answer	• some used $\frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}$, which works for (i) but not (ii)
(ii) $P(\text{exactly } 2W) = \dfrac{{}^4C_2 \cdot {}^9C_1}{{}^{13}C_3} = \dfrac{6\times 9}{286} = \dfrac{27}{143}$	① answer	• for (ii), the issue is combinations so $\frac{4}{13}\times\frac{3}{12}\times\frac{9}{11}$ gives $\frac{9}{143}$ which is one permutation only (this was a common error)
(b)		Note: (b), (c) and (d) - except for c(iii) - are all Show questions, which were universally poorly don[e]
(i) Δ's similar $\Rightarrow$ sides in ratio i.e. $\dfrac{SP}{TQ} = \dfrac{SN}{TN}$ P on circle $\Rightarrow P(\cos\theta, \sin\theta)$ $\therefore SN = 2$ (diameter of circle) $TQ = \cos\theta$ $TN = SN - OT$ $= 1 - \sin\theta$ $\therefore\ SP = \dfrac{SN.TQ}{NN}$ $= \dfrac{2.\cos\theta}{1-\sin\theta}$ as reqd.	① components explained/derived ① ratio + algebr[a] correct	You MUST explain where each component comes from!! • (i) explicitly stated "Using the fact that...", so students who did not use this fact got no ma[rks]
(ii) $LHS = \dfrac{\cos\theta}{1-\sin\theta}$ $= \dfrac{\cos\theta}{1-\sin\theta} \times \dfrac{1+\sin\theta}{1+\sin\theta}$ $= \dfrac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$ $= \dfrac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$ $= \dfrac{1+\sin\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ $= \sec\theta + \tan\theta$ $= RHS$	① manipulations correct	• many poor attempts at the trig manipulations - many students unfamiliar with trig identities in general.
(iii) $\angle SOQ = \frac{\pi}{2} + \theta$ $\angle SNQ = \frac{1}{2}\angle SOQ$ ($\angle$ at circ $= \frac{1}{2}\angle$ at centre) $= \frac{1}{2}\left(\frac{\pi}{2}+\theta\right)$ $= \frac{\theta}{2} + \frac{\pi}{4}$ as reqd.	① use of circle property (or equivalent)	• some students used isosceles triangle properties correctly to receive the mark. • many poor attempts that missed circle properties.

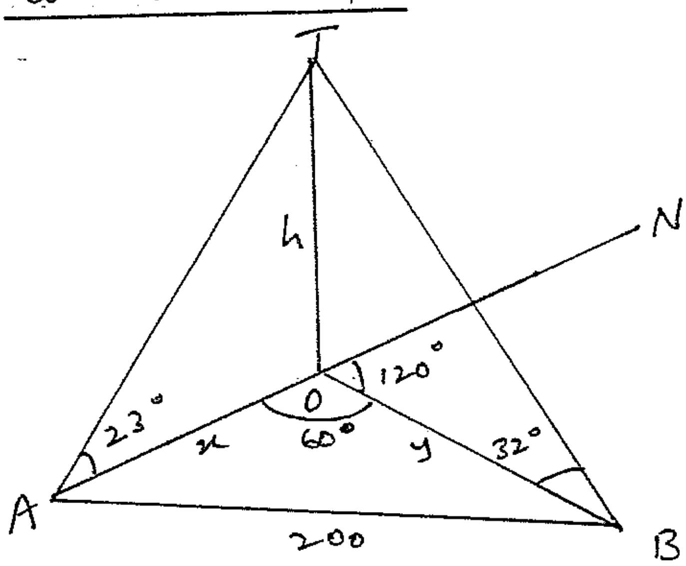
(b)	marking	Comments
(iv) $\tan\left(\frac{\theta}{2}+\frac{\pi}{4}\right)$ $= \tan(\angle SNP)$ from(ii) $= \frac{SP}{SN}$ (tan defn in ΔSNP) $= \frac{2\cos\theta}{1-\sin\theta} \times \frac{1}{2}$ (from(i)) $= \frac{\cos\theta}{1-\sin\theta}$ $= \sec\theta + \tan\theta$ (from (iii))	① linking all the components together.	• very few able to follow and explain all links
(c)		
(i) $T = x + Ae^{-kt}$ so $Ae^{-kt} = T - x$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T-x)$	① full explanation	• many did $-k(Ae^{-kt}+x-x)$ • many also wrote: $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(T-x)$. which was awarded no marks – all this info already given, so nothing was "shown" in this working
(ii) $T = x + Ae^{-kt}$ $t=0,\ T=18$ (initially) $\therefore 18 = x + Ae^{0}$ $= x + A$ $\therefore A = 18 - x$	① steps to A value correct	• (ii) was probably the best done part of Q13, although several students skipped steps and lost marks.
so $T = x + (18-x)e^{-kt}$ then $t = \frac{15}{2},\ T = 10$ thus $10 = x + (18-x)e^{-\frac{15}{2}k}$ $10 - x = (18-x)e^{-\frac{15}{2}k}$ $\frac{(10-x)}{(18-x)} = e^{-\frac{15}{2}k}$ $-\frac{15}{2}k = \ln\left(\frac{10-x}{18-x}\right)$	① work preparatory to taking logs	
$\therefore k = -\frac{2}{15}\ln\left(\frac{10-x}{18-x}\right)$ Thus $T = x + (18-x)e^{-\frac{-2t}{15}\ln\left(\frac{10-x}{18-x}\right)}$ $= x + (18-x)e^{\frac{2t}{15}\ln\left(\frac{10-x}{18-x}\right)}$ as reqd.	① k value correct and correctly substituted	
(iii) $x = 5,\ t = 15$ then $T = 5 + (18-5)e^{\frac{2}{15}\left[\ln\left(\frac{10-5}{18-5}\right)\right]15}$ $= 5 + 13e^{2\ln\left(\frac{5}{13}\right)}$ $\doteqdot 6.92307\ldots$ $\doteqdot 7°$ (nearest °)	① answer	• many correctly substituted but could not enter into their calculator correctly! • Read! to the nearest degree!

	marking	comments.
d) Show $4^n-1-7n>0$ for $n\geq 2$		• in steps 1 and 3, the LHS = .. = = RHS process is needed. Many students could not do this successfu
1. Show true for $n=2$.		
$LHS = 4^2-1-7\times 2$		
$= 1$		
>0		
Hence true for $n=2$		
2, Assume true for $n=k$		• most students were ab to get to this point and gain this mark.
ie assume $4^k-1-7k>0$	① correct assumption	
or $4^k>7k+1$	plus previous working	
3. Show true for $n=k+1$		• most wrote few, if any, of the supporting "words" needed to explain what they were doing.
ie show $4^{k+1}-1-7(k+1)>0$		
$LHS = 4^{k+1}-1-7(k+1)$		
$= 4.4^k-1-7k-7$		
$> 4(7k+1)-8-7k$ using assumption	① correct use of assumption	• for step 3, the line $4^{k+1}-1-7(k+1)>0$ was the starting point for many students' attempts. This is a statement of truth, which we are trying to <u>show</u>, so cannot be used. Most got no marks from this poin on.
$> 28k+4-8-7k$		
$> 21k-4$		
>0 as $k\geq 2$ means $21k-4>38$	① resolved for >0 and	
Hence true for $n=k+1$	concluding statement	
4. But as true for $n=2$, hence true for $n=2$, and so on by principle of induction.		
		• $4^{k+1}-1-7k>0$ was also common for 3.
		• clearly show the use of the assumption at which point the sign becomes $>$ and stays that way !!

Note: Q13 solns reduced from 4 to 3 pages - no page 10.

Question 14

i)



$\tan 23° = \frac{h}{x}$

$x = h \cot 23°$

$\tan 32° = \frac{h}{y}$

$y = h \cot 32°$

ii. Using cosine rule:

$$200^2 = x^2 + y^2 - 2xy \cos 60°$$

$$= h^2 \cot^2 23° + h^2 \cot^2 32° - 2h^2 \cot 23 . \cot 32° \times \cos 60°$$

$$= h^2 [(\cot^2 23° + \cot^2 32° - (\cot 23° . \cot 32°)]$$

$$h^2 = \frac{200^2}{\cot^2 23° + \cot^2 32° - \cot 23° \cot 32°}$$

$$h^2 = \frac{200^2}{(5.55 + 2.56 - 2.36 \times 1.6)}$$

$$= 9208$$

$h = 96$ m (nearest m).

Comment: i) most students were given the mark allocated for the diagram. However, it was poorly drawn. Diagrams should be at least $\frac{1}{4}$ of a page big. A ruler should have been used and the given information should have been labelled clearly.

ii) Some need to learn the cosine rule. Many did not use the version of the cosine rule for finding a side. This resulted in errors because making h^2 the subject became harder.

b) i)

$$\ddot{y} = -g$$

$$\dot{y} = -\int g \, dt$$

$$= -gt + c_1, \quad t = 0, \ \dot{y} = 0$$

$$\therefore c_1 = 0$$

$$y = -\int gt \, dt$$

$$= \frac{-gt^2}{2} + c_2, \quad \text{at } t = 0, \ y = h$$

$$\therefore c_2 = h$$

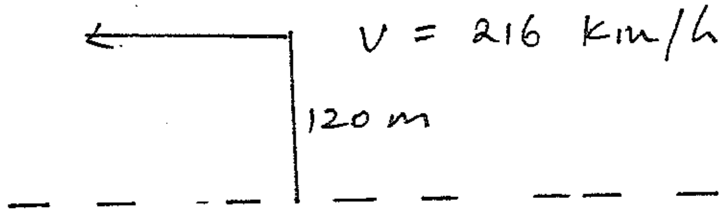
$$\therefore y = h - \frac{gt^2}{2}$$

Comments

b) i) Poorly done. This is a 'show' question. Students needed to show the integration process and obtain the values of the constants of integration with reasoning. $\therefore C = 0$

$x = vt$ was given $\therefore$ did not need to show this.

ii)



$$v = 216 \text{ km/h} = 60 \text{ m/s}$$

The canister hits the water when $y = 0, h = 120$

$$\therefore \quad 0 = 120 - \frac{10}{2}t^2 \quad ✓$$

$$5t^2 = 120$$

$$t = 4.9 \text{ sec.} \quad ✓$$

ii) Overall, well done. Question asked for answer to 1 dec. pl. but marks were not deducted if you didn't do this.

iii) After 4.9 sec, the helicopter has travelled:

$$x = vt$$

$$= 60 \times 4.9 \quad ✓$$

$$= 294 \text{ m from the origin}$$

$$V_{sailor} = 3.6 \text{ km/h}$$

$$= 1 \text{ m/s} \quad ✓$$

$$\therefore x_{sailor} = 1 \text{ m/s} \times 4.9 \text{ sec}$$

$$\text{Sailor's drift} = 4.9 \text{ m} \quad ✓$$

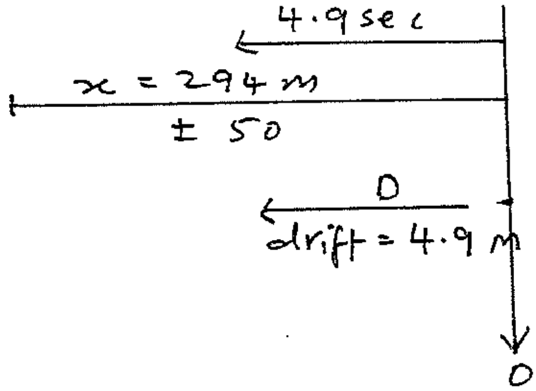
iii) Many were not successful with this question. A major error was not being consistent with units. Most had no idea how to begin. Marks were strictly allocated as shown here.

Comments

$\therefore$ Sailor's distance from O is :
$$D + 4 \cdot 9$$

$$294 - 50 \le D + 4 \cdot 9 \le 294 + 50$$

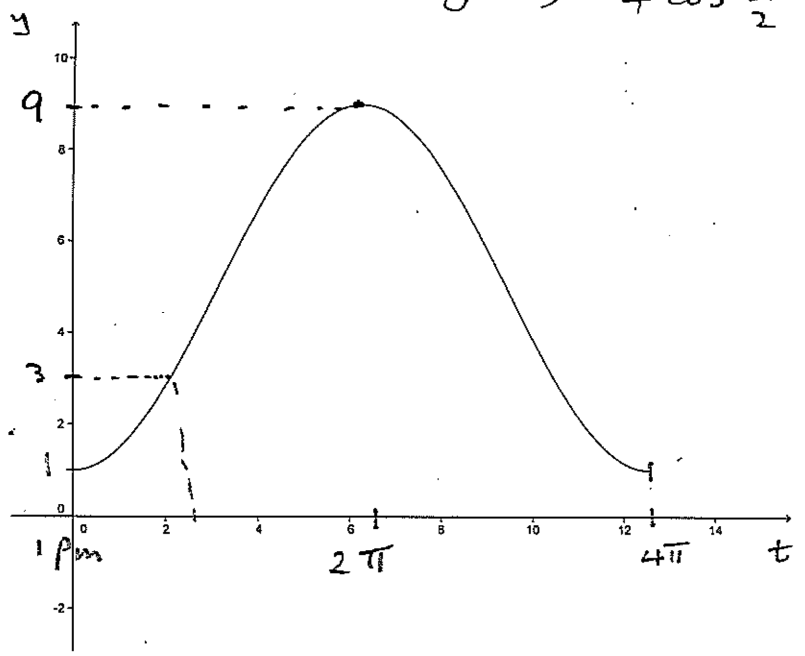
$$239 \cdot 1 \le D \le 339 \cdot 1 \quad ✓$$



c.

$$y = 5 - 4 \cos \frac{t}{2}$$

i)



✓ shape
✓ features

c)i) Graph was unsatisfactory. Axes should be labelled. The 'period' was wrong. Extreme points were not labelled. The restricted domain was not observed. Graph in wrong direction, etc.

c)ii) Too many careless errors eg. writing am, instead of p.m., $2 \times \frac{\pi}{3} = \frac{\pi}{6}$, etc. Time is not in degrees & minutes. Should have been working in radians.

ii

$$3 = 5 - 4 \cos \frac{t}{2}$$
$$4 \cos \frac{t}{2} = \frac{1}{2}$$
$$\frac{t}{2} = 1 \cdot 0475$$
$$t = 2 \cdot 09 \text{ h} \quad ✓$$
$$= 2 \text{ h } 6 \text{ mins}$$

$\therefore$ the earlies time the ship can enter the creek is 3.06 PM ✓

END